Validation test No. 1

1. The aim of the test

The purpose of the test is to verify the correctness of the pressure drop and velocity profile calculations in a rectilinear square channel at a laminar flow regime.

2. Theoretical basis

One-dimensional flow of an incompressible fluid in a smooth circular pipe forms the basis for the pressure drop analysis in internal flows. The following equation is derived based on the continuum assumption for Newtonian liquid flows in minichannels and microchannels. (Kandlikar, 2013)

The frictional pressure drop Δp [Pa] over a circular channel of length L [m] and diameter D [m] can be obtained from the equation of Darcy–Weisbach type:

$$\Delta p = \frac{2f\rho u_m^2 L}{D} = 2(f\,\mathrm{Re})\frac{u_m L \mu}{D^2} = 2\,Po\frac{u_m L \mu}{D^2} \quad (1)$$

where:
u_m – is the mean flow velocity in the channel [m/s],
ρ – is the fluid density [kg/m^3],
μ – is fluid dynamic viscosity [Pa s],
Re – Reynolds number, $\mathrm{Re} = \frac{u_m D_h \rho}{\mu}$
Po – Poiseuille number.

The Fanning friction factor f [-] in Eq. (1) depends on the flow conditions, the channel wall geometry, and surface conditions:

1. laminar or turbulent flow,
2. flow-channel geometry,
3. fully developed or developing flow,
4. smooth or rough walls.

For noncircular flow channels, the D in Eq. (1) is replaced by the hydraulic diameter D_h [m], represented by the following equation:

$$D_h = \frac{4A_c}{P_w} \quad (2)$$

were:
A_c - is the flow-channel cross-sectional area [m^2],
P_w - is the wetted perimeter [m].

For a rectangular channel of sides *a* and *b*, D_h is given by:

$$D_h = \frac{4ab}{2(a+b)} = \frac{2ab}{(a+b)} \quad (3)$$

The pressure drop in the straight channel can be also calculated from the Hagen-Poiseuille equation for the fully developed laminar flow in a smooth circular pipe:

$$\Delta p = \frac{8\mu L Q}{\pi R^4} = \frac{8\pi\mu L Q}{A^2} \quad (4)$$

were:

Δp is the pressure difference between the two channel ends [Pa],
L is the length of pipe [m],
μ is the dynamic viscosity [Pa*s],
Q is the volumetric flow rate [m^3/s],
R is the pipe radius [m],
A is the cross section of pipe [m^2].

The Hagen-Poiseuille equation can be obtained from the Darcy–Weisbach equation when we take into account that the Fanning friction factor f [-] in Eq. (1) is related to Re (Reynolds) and Po (Poiseuille) numbers as follows:

$$f = \frac{Po}{Re} \qquad (5)$$

For circular pipe

$$Po = fRe = 16 \qquad (6)$$

Shah and London (Shah & London, 1978) provided the following equation for a rectangular channel with short side a and long side b, and a channel aspect ratio defined as $\alpha_c = a/b$:

$$fRe = 24(1 - 1.3553\alpha_c + 1.9467\alpha_c^2 - 1.7012\alpha_c^3 + 0.9564\alpha_c^4 - 0.2537\alpha_c^5) \qquad (7)$$

For rectangular channels and fully developed laminar flow in smooth ducts, Kakac et al. (Kakaç, Shah, & Aung, 1987) derived values of $Po = f\,Re$ for several aspect ratios:

Table 1. Po values for rectangular ducts, Kakac et al. 1987

		$Po = f\,Re$
Rectangular, aspect ratio, b/a =	1	14.23
	2	15.55
	3	17.09
	4	18.23
	6	19.70
	8	20.58
	∞	24.00

M. Serwinski (Serwiński, 1971) presents a little bit different values for a $= \lambda\,Re = 4\,f\,Re$:

Table 2. Parameter a values for several shapes, M. Serwinski 1971

Duct Shape	b/a ratio	a
circle, diameter a		64
square, side a		57
equilateral triangle, side a		53

ring, distance a		96
Rectangular, a/b	1/∞	96
	0.1	85
	0.2	76
	0.25	73
	0.33	69
	0.5	62

P. Panigrahi (Panigrahi, 2016) presents more precise values for a $= \lambda$ Re = 4 *f* Re:

Table 3. Parameter a values for several shapes, P. Panigrahi 2016

h/w	$f \cdot Re$
1.0	56.880
0.9	57.048
0.8	57.510
0.6	59.920
0.4	65.470
0.2	76.280
0.1	84.640

Franck Delplace (Delplace, 2018) in his article presented most curate value of ***f/2*Re* = 7.11353554** for square duct based on analytical solution of Navier-Stokes (NS) equation.

2.1 Developing laminar flow

As flow enters a duct, the velocity profile begins to develop along its length, ultimately reaching the fully developed Hagen-Poiseuille velocity profile. Almost all the analyses available in the literature consider a uniform velocity condition at the inlet. The length of the hydrodynamic developing region L_h for circular duct is given by the following well-accepted equation (Kandlikar, 2013):

$$\frac{L_\mathrm{h}}{D_\mathrm{h}} = 0.05Re \qquad (8)$$

Apparent friction factor f_{app} accounts for the pressure drop due to friction and the developing region effects. It represents an average value of the friction factor over the flow length between the entrance section and the location under consideration. Thus the pressure drop in a channel of hydraulic diameter D_h over a length x from the entrance is expressed as (Kandlikar, 2013):

$$\Delta p = \frac{2f_\mathrm{app}\rho u_\mathrm{m}^2 x}{D_\mathrm{h}} \qquad (9)$$

The difference between the apparent friction factor over a length x and the fully developed friction factor *f* is expressed in terms of an incremental pressure defect K(x):

$$K(x) = (f_\mathrm{app} - f)\frac{4x}{D_\mathrm{h}} \qquad (10)$$

For x > L_h the incremental pressure defect attains a constant value K(∞), known as Hagenbach's factor. Combining Eqs. (9) and (10), the pressure drop can be expressed in terms of the incremental pressure drop:

$$\Delta p = \frac{2(f_{\mathrm{app}}Re)\mu u_{\mathrm{m}}x}{D_{\mathrm{h}}^2} = \frac{2(fRe)\mu u_{\mathrm{m}}x}{D_{\mathrm{h}}^2} + K(x)\frac{\rho u_{\mathrm{m}}^2}{2} \quad (11)$$

Chen (1972) proposed the following equation for K(∞) for the circular geometry:

$$K(\infty) = 1.20 + \frac{38}{Re} \quad (12)$$

The nondimensionalized length x^+ is given by:

$$x^+ = \frac{x/D_{\mathrm{h}}}{Re} \quad (13)$$

By considering the rectangular channels as a subset of the trapezoidal geometry, (Kandlikar, 2013) obtained the following curve-fit equation for the Hagenbach's factor for rectangular channels:

$$K(\infty) = 0.6796 + 1.2197\alpha_{\mathrm{c}} + 3.3089\alpha_{\mathrm{c}}^2 - 9.5921\alpha_{\mathrm{c}}^3 + 8.9089\alpha_{\mathrm{c}}^4 - 2.9959\alpha_{\mathrm{c}}^5 \quad (14)$$

The constant 0.05 in Eq. (8) is modified to around 1 for fully developed value in rectangular ducts and microflows. Table 4, derived from Phillips (1987) (Kandlikar, 2013), gives the values of the apparent friction factor in tabular form.

*Table 4. Values of f_{app}*Re = Po_{app} for square duct*

$x^+ = (x/D_h)/Re$	$\alpha_c = 1.0$
0	142.0
0.001	111.0
0.003	66.0
0.005	51.8
0.007	44.6
0.009	39.9
0.01	38.0
0.015	32.1
0.02	28.6
0.03	24.6
0.04	22.4
0.05	21.0
0.06	20.0
0.07	19.3
0.08	18.7
0.09	18.2
0.10	17.8
0.20	15.8
> 1.0	14.2

For intermediate values use the curve-fit equations provided below (self-determined values).

$$y = y_0 + A_1 * \exp\left(\frac{-(x-x_0)}{t_1}\right) + A_2 * \exp\left(\frac{-(x-x_0)}{t_2}\right) + A_3 * \exp\left(\frac{-(x-x_0)}{t_3}\right) \quad (15)$$

where:

y_0 14.23298

x_0 -0.00761

A_1 8.64341
t_1 0.1232
A_2 2903.24361
t_2 0.00221
A_3 44.49058
t_3 0.016

y = f_{app}*Re
x - is the distance from the beginning of channel [m],

Moreover, at the outlet of the channel there are also small deviations from the linear relationship between the pressure drop and the length of the channel. See the Kandlikar's book for more information.

2.2 Velocity profile in a square duct

The fully developed velocity profile for rectangular ducts has been determined using an analogy with the stress function of the theory of elasticity (Shah & London, 1978). The velocity profile, provided by the solution of NS equation with the Dirichlet v = 0 boundary condition at the wall surface can be calculated from equation:

$$u = -\frac{16c_1a^2}{\pi^3}\sum_{n=1,3,\ldots}^{\infty}\frac{1}{n^3}(-1)^{(n-1)/2}\left[1-\frac{\cosh(n\pi y/2a)}{\cosh(n\pi b/2a)}\right]\cos\left(\frac{n\pi z}{2a}\right)$$
$$u_{\mathrm{m}} = -\frac{c_1a^2}{3}\left[1-\frac{192}{\pi^5}\left(\frac{a}{b}\right)\sum_{n=1,3,\ldots}^{\infty}\frac{1}{n^5}\tanh\left(\frac{n\pi b}{2a}\right)\right] \quad (16)$$

where the channel aspect ratio is defined as:

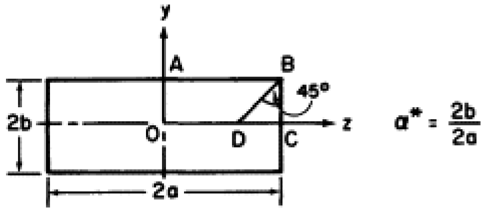

Figure 1. Case geometry

M. Spiga and G.L. Morini (Spiga & Morino, 1994) proposed the simplest form of solution of NS for rectangular duct describing a velocity profile that is derived using the finite Fourier transform:

$$V(x,y) = \frac{16\beta^2}{\pi^4}\sum_{n\,odd}^{\infty}\sum_{m\,odd}^{\infty}\frac{\sin(n\pi\frac{\xi}{a})\sin(m\pi\frac{\eta}{b})}{nm(\beta^2n^2+m^2)} \quad (17)$$

were:

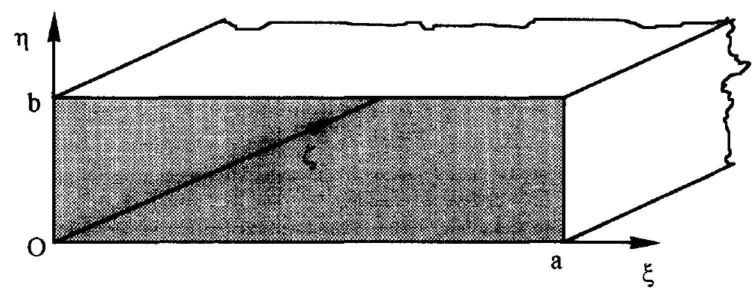

Figure 2. Case geometry

The dimensionless coordinates are $x=\xi/a$ ($0\leq x\leq l$) and $y=\eta/a$ ($0\leq y\leq\beta$, being β the aspect ratio b/a). The dimensionless velocity is defined as $V(x,y) = v(x,y)/P$, where:

$$\mathbf{P} = \frac{\mathbf{a}^2}{\mu}\left(-\frac{\partial p}{\partial \zeta} + \rho g_\zeta\right) \quad (18)$$

3. Geometry description

The test geometry is a straight square channel (rectangular cuboid) of length [lu] x = 1000, y = 20, z = 20. Number of points X = 1001, Y = 21, Z = 21 (computational nodes = 441 441).
The physical dimension of channel is 0.5x0.01x0.01 [m], range x,y,z = [0:0.5, 0:0.01, 0:0.01].
The geometry is presented in Figures 3-5.

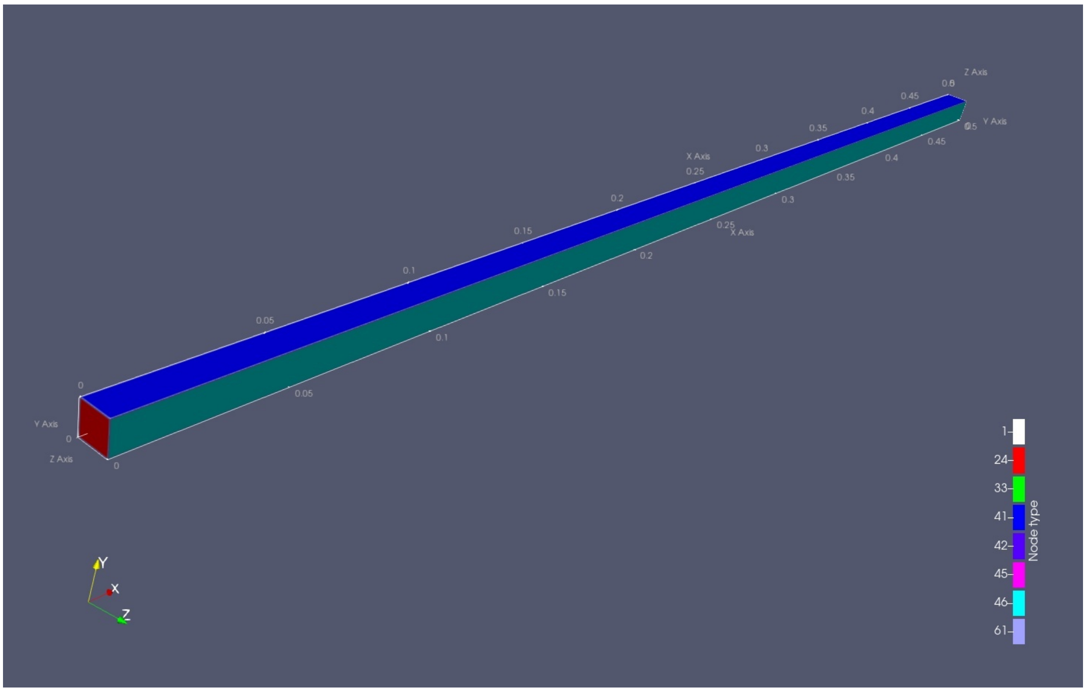

Figure 3. The geometry of test case

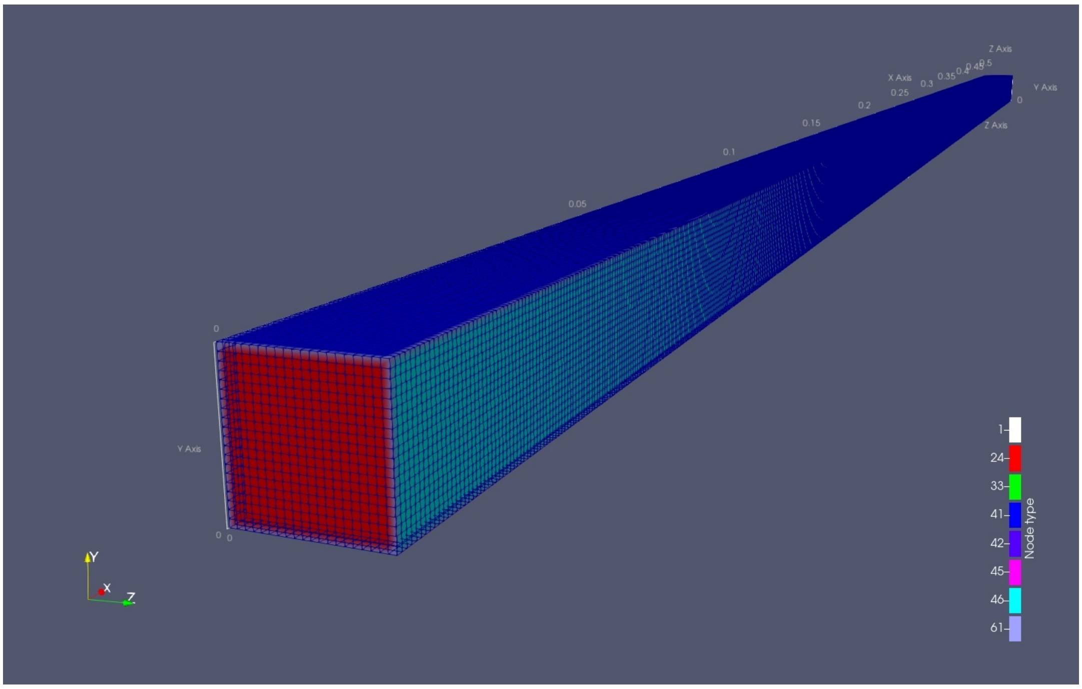

Figure 4. The grid

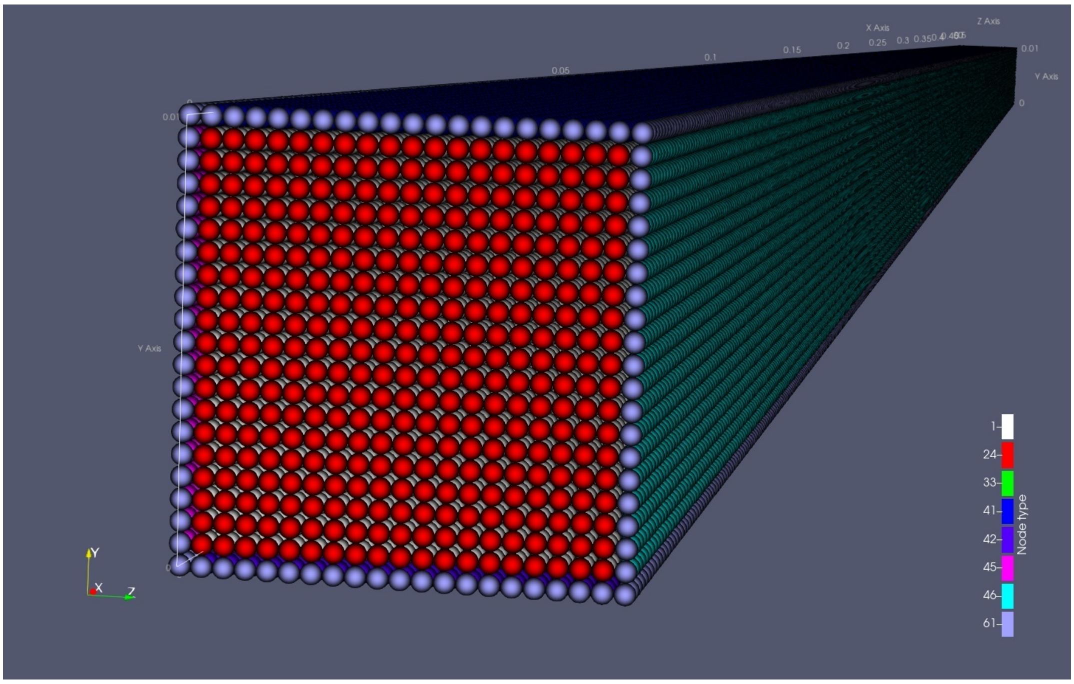

Figure 5. Computational nodes

Grid statistics:
Total number of computational nodes = 441441
Number of solid (0) nodes = 0
Number of fluid (1) nodes = 360639
Number of bounce back (61) nodes = 4156
Number of velocity (20) nodes = 361
Number of velocity0 (40) nodes = 75924
Number of pressure (30) nodes = 361
Number of periodic (4) and (5) nodes = 0
Ratio of non-fluid nodes to total number of computational nodes [%] = 18.3041
Number of MFThreads = 10

All case files are placed in folder:
Data/Validation_tests/Case_MEDIUM_straight_square_channel/geometry/
Geometry files:
geometry.stl (geometry STL file from OpenSCAD)
X1000.csv (text file with coordinates of nodes placed on outlet surface)
X0.csv (text file with coordinates of nodes placed on inlet surface)

4. Case and threads parameters

Table 5. Simmulation parameters

Parameter name	Parameter value
Solver:	
Microflow 3D	0.26
Boundary conditions:	
velocity inlet (Dirichlet type) on inlet [m/s]	[x = 0.001, y = 0, z = 0]
pressure outlet on outlet (Neumann type), Δp [Pa]	0
velocity "0" (Dirichlet type) on walls	0
full way bounce-back on edges and corners	0
Initial conditions (t = 0) - equilibrium model:	
everywhere in fluid velocity [m/s]	[0,0,0]
Fluid model:	
incompressible flow model	
physical fluid density [kg/m3]	1000
physical fluid kinematic viscosity [m^2/s]	1e-6
Collision model:	
BGK	
Other solution parameters:	
tau	1.0
lattice type	D3Q19
physical characteristic length [m]	0.01
physical characteristic mean velocity for characteristic dimension [m/s]	0.0009025
Termination conditions (both conditions must be met simultaneously):	
Scaled velocity residue [-]	1.0e-6
Mass flow error [%]	0.1

For more details on implemented boundary conditions, collision and fluid models refer to (Hecht & Harting, 2010), (Zou & He, 1997) and (Guo & Shu, 2013).

All configuration files are placed in folder:
Data/Validation_tests/Case_MEDIUM_straight_square_channel/params/

case_params.cfg file :

```
# Scope for Microflow 3D v0.1
MF01
{

GeometryParams
{
GeometryName = "Straight square channel"; # Name of the geometry
```

VoxelSize = "0.05"; # Length of edge of square voxel (unite volume). E.g if the length of geometry edge equals 10 units and we want to have 11 nodes on the edge, the voxel size is 1.
PhysicalVoxelSize = "0.5e-3"; # Length in [m] that corresponds to 1 Lu (lattice unit of length) or one voxel edge length (dx_phys).
AddInnerWall = "0"; # Adds additional layer of points on inner walls when substracting small closed surfaces from the grid. The value should be < VoxelSize to add one voxel of fluid type. (0 = no addition).
}

BasicParams
{
GeometryDefinition = "FromSTL"; # (FromSTL) Geometry definition method: 0 - FromSTL.
FluidFlowModel_MT = "Incompressible"; # (QuasiCompressible/Incompressible) Flow models: 0 - quasi compressible (e.g air), 1 - incompressible (e.g water) - Zou et al. 1995.
CollisionModel_KT = "BGK"; # (BGK/MRT/MRT2/FBGK) Collision models: 0 – BGK, 1 - MRT standard, 2 - FBGK (BGK with force), 3 - MRT with optimized parameters for stability.
LatticeType_La = "D3Q19"; # (D3Q19) Lattice type: 0 - D3Q19
}

PhysicalParams
{
FluidPhysicalViscosity_Nu_Phys = "0.000001"; # Physical fluid kinematic viscosity [m^2/s] (e.g 1.51E-5 air, 1E-6 water).
FluidPhysicalReferenceDensity_Rho0_Phys = "1000"; # Physical fluid density [kg/m3] (e.g 1.2 air, 1000 water).
Tau = "1"; # Tau > 0.5, minimal stable values: 0.520 MRT, 0.58 BGK, 0.504 MRT2.
XYZForce_fiz = ["0.0","0.0","0.0"]; # The component X, Y, Z of the volumetric force, e.g. gravitational force [N/m3] (body force) F = rho * g (g = 9.81).
}

CharactersticParams
{
CharactLengthPhysical_L_CH_Phys = "0.010"; # Physical characteristic length [m].
CharactVelocityPhysical_U_CH_Phys = "0.0009025"; # Physical characteristic mean velocity for characteristic dimension [m/s].
}

TerminationCondition
{
Sc_VelocityResidueError_ErrV = "1.0e-6"; # Velocity termination condition - the mean scaled residue of average velocity for subsequent steps. (0 - not used)
MassFlowError_ErrM = "0.1"; # Mass flow termination condition - the mass flow error for subsequent steps calculated for balance surfaces [%]. (0 - not used)
}

InitialCondition
{
ReferenceDensityLB_Rho0_LB = "1.0"; # LB density of fluid at the reference point, usually at the outflow.
XYZInitialVelocityPhysical_U0 = ["0","0","0"]; # Initial value of LB velocity, vector [X,Y,Z].
}

DefaultBoundaryCondition
{
DefaultWallNode_BN = "40"; # Default type of nodeForAutoThreading on the wall (40)
DefaultNode_NN = "61"; # Default edge/corner/unrecognized nodeForAutoThreading (61)
}

RWParams
{
CheckPointNumberRead_CHP_R = "0"; # Check point number for resuming calculations, 0 - no check point resuming.
VTKWriteStep_VTK_W = "0"; # Step for save check points and VTK/VTI data files, 0 - write data only at the begining and at the end of calculations.
VTKFileMaxNumber_VTK_Max = "2"; # The maximum number of VTK/VTI files stored on the disk, in addition to initial and final files.
ConsoleWriteStep_K_W = "100"; # The step for displaying information on the console.
}

BalanceParams
{
AutomaticSurfaceChoice = "false"; # The method of choosing balancing surfaces. If "true", the mass balance is calculated for all surfaces type 20 (velocity inlet) and 30 (pressure outlet). If "false", it is necessary to define the balance surfaces in the thread.cfg file. (true/false)
}

ThreadParams
{
MFThreadMaxSize = "0"; # The maximal number of nodes grouped in one MFThread. Should be less than 4 294 967 296; 0 - automatic detection (assigned max. allowable number).
}

VTKFileParams
{
NodeID_Save = "true"; # Does the nodeType have to be written to the file (true/false)?
NodeUidThread_Save = "false"; # Does the uidTread have to be written to the file (true/false)?
NodeType_Save = "true"; # Does the nodeType have to be written to the file (true/false)?
NodeCoordinate_Save = "true"; # Does the nodeForAutoThreading coordinates have to be written to the file (true/false)?

VelocityLB_Save = "true"; # Does the LB fluid velocity [-] have to be written to the file (true/false)?
VelocityPhys_Save = "true"; # Does the physical fluid velocity [m/s] have to be written to the file (true/false)?

RhoLB_Save = "true"; # Does the rho LB [-] have to be written to the file (true/false)?
RhoPhys_Save = "true"; # Does the fluid density [kg/m3] have to be written to the file (true/false)?

PressurePhys_Save = "true"; # Does the physical pressure [Pa] have to be written to the file (true/false)?

InitialRhoLB_Save = "false"; # Does theinitial/boundary value of rho LB have to be written to the file (true/false)?
InitialVelocityLB_Save = "false"; # Does the initial/boundary value of LB velocity have to be written to the file (true/false)?

FQ19_Save = "false"; # Does the MFQ19 values have to be written to the file (true/false) ?

PropagationDirections_Save = "false"; # Does the nodeForAutoThreading propagation directions for each MFQ19 direction have to be written to the file (true/false)?

ThreadsNamed_Save = "true"; # Does the named set of points (threads) have to be written to the file (true/false)?
}

CPUParams
{
CPU_ThreadsNr = "6"; # Number of CPU threads to be allocated (0 = max available number)
}

} # End of scope for MF01

thread_params.cfg file:

MF01
{

uid-Thread
{
ThreadName = "inlet";
NodeTypeID = "20"; # Dirichlet velocity boundary condition.
GridType = "VDB"; # VDB/VTI point coordinates (VDB).
NodeCoordinateFileColumnNames = ["MF NodeCoord:0","MF NodeCoord:1","MF NodeCoord:2"]; # Column names for coordinates x,y,z in the specified file.
NodeFilePath = "X0.csv"; # Points .csv type file name and path.
Velocity_fiz = ["0.001","0","0"]; # [m/s]
IsInletBalanceSurface = "true"; # Sets the surface type outlet for manual calculations of mass balance printed on console. (true/false)
}

uid-Thread
{
ThreadName = "outlet";
NodeTypeID = "30"; # Pressure boundary condition.
GridType = "VDB"; # VDB/VTI point coordinates (VDB).
NodeCoordinateFileColumnNames = ["MF NodeCoord:0","MF NodeCoord:1","MF NodeCoord:2"]; # Column names for coordinates x,y,z in the specified file.
NodeFilePath = "X1000.csv"; # Points .csv type file name and path.
Pressure_fiz = "0"; # Relative pressure to reference pressure [Pa]
IsOutletBalanceSurface = "true"; # Sets the surface type inlet for manual calculations of mass balance printed on console. (true/false)
}

} # End of scope for MF01

5. Solution logs

```
-------------------------------------------------------------------------------------------------------------------
Microflow 3D v0.26
-------------------------------------------------------------------------------------------------------------------
Microflow framework >>> Initialization info:
Parsed file: -----> Config/Microflow.cfg
Parsed file: -----> Data/Validation_tests/Case_MEDIUM_straight_square_channel/params/case_params.cfg
Parsed file: -----> Data/Validation_tests/Case_MEDIUM_straight_square_channel/params/thread_params.cfg
-------------------------------------------------------------------------------------------------------------------
Microflow framework >>> Geometry building info:
Grid from Data/Validation_tests/Case_MEDIUM_straight_square_channel/geometry/geometry.stl has been read successfully ----> OpenSCAD_Model
441 data values are read from the file: Data/Validation_tests/Case_MEDIUM_straight_square_channel/geometry/X0.csv
441 data values are read from the file: Data/Validation_tests/Case_MEDIUM_straight_square_channel/geometry/X1000.csv
-------------------------------------------------------------------------------------------------------------------
MFSolver_CPU >>> Case and program parameters initialization info:
-------------------------------------------------------------------------------------------------------------------
MFDatabase >>> Threads building info:
-------------------------------------------------------------------------------------------------------------------
MFSolver_CPU >>> Threads data initialization info:
-------------------------------------------------------------------------------------------------------------------
MFSolver_CPU >>> Main loop info:

Re = 9.025
U_Ch_LB = 0.0752083
Dt_phys [s] = 0.0416667
L_ch_LB [Lu] = 20
Nu_LB = 0.166667
Fx_LB = 0
Fy_LB = 0
Fz_LB = 0

Grid statistics:
Total number of computational nodes = 441441
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Number of fluid (1) nodes = 360639
Number of bounce back (61) nodes = 4156
Number of velocity (20) nodes = 361
Number of velocity0 (40) nodes = 75924
Number of pressure (30) nodes = 361
Number of periodic (4) and (5) nodes = 0
Ratio of non-fluid nodes to total number of computational nodes [%] = 18.3041
Number of MFThreads = 10

CPU info:
Maximum number of available threads = 8
Maximum number of available processor cores (including virtual) = 8
Number of CPU threads allocated for solver = 6

VTI file was saved successfully: ----> Data/Validation_tests/Case_MEDIUM_straight_square_channel/output/microflow_output0.vti

Table legend: Sc = Scaled, M = Mean, MLUPS = Million of Lattice nodes Updated Per Second.
==================================================================================================================================================================
| Step    | Sc velocity residue, [-] | Maximal LB velocity, [-] | M outlet velocity, [m/s]  | M inlet velocity, [m/s]  | Error mass flow, [%] | Error mass flow LB, [%] | MLUPS, [-]            |
==================================================================================================================================================================
| 0       | DeltaV = inf             | uLB_maks = 0.000000e+00  | OutVMean = 0.000000e+00   | InVMean = 9.002494e-04   | DMF = 1.000000e+02   | DMF_LB = 1.000000e+02   | MLUPS = 0.000000e+00   |
| 1000    | DeltaV = 2.000000e-03    | uLB_maks = 1.478477e-01  | OutVMean = 0.000000e+00   | InVMean = 9.002494e-04   | DMF = 1.000000e+02   | DMF_LB = 1.000000e+02   | MLUPS = 4.524423e+01   |
| 2000    | DeltaV = 3.521882e-04    | uLB_maks = 1.501586e-01  | OutVMean = 3.910126e-07   | InVMean = 9.002494e-04   | DMF = 9.995657e+01   | DMF_LB = 9.995657e+01   | MLUPS = 4.530781e+01   |
| 3000    | DeltaV = 2.041890e-04    | uLB_maks = 1.512955e-01  | OutVMean = 1.964373e-05   | InVMean = 9.002494e-04   | DMF = 9.781797e+01   | DMF_LB = 9.781797e+01   | MLUPS = 4.528599e+01   |
| 4000    | DeltaV = 1.442360e-04    | uLB_maks = 1.520016e-01  | OutVMean = 5.789760e-05   | InVMean = 9.002494e-04   | DMF = 9.356872e+01   | DMF_LB = 9.356872e+01   | MLUPS = 4.531203e+01   |
| 5000    | DeltaV = 1.114831e-04    | uLB_maks = 1.524999e-01  | OutVMean = 1.043162e-04   | InVMean = 9.002494e-04   | DMF = 8.841252e+01   | DMF_LB = 8.841252e+01   | MLUPS = 4.530752e+01   |
| 6000    | DeltaV = 9.072751e-05    | uLB_maks = 1.528804e-01  | OutVMean = 1.529177e-04   | InVMean = 9.002494e-04   | DMF = 8.301385e+01   | DMF_LB = 8.301385e+01   | MLUPS = 4.530941e+01   |
| 7000    | DeltaV = 7.625142e-05    | uLB_maks = 1.532037e-01  | OutVMean = 2.008286e-04   | InVMean = 9.002494e-04   | DMF = 7.769189e+01   | DMF_LB = 7.769189e+01   | MLUPS = 4.530603e+01   |
| 8000    | DeltaV = 6.544956e-05    | uLB_maks = 1.534812e-01  | OutVMean = 2.467529e-04   | InVMean = 9.002494e-04   | DMF = 7.259061e+01   | DMF_LB = 7.259061e+01   | MLUPS = 4.531349e+01   |
| 9000    | DeltaV = 5.699601e-05    | uLB_maks = 1.537277e-01  | OutVMean = 2.901662e-04   | InVMean = 9.002494e-04   | DMF = 6.776824e+01   | DMF_LB = 6.776824e+01   | MLUPS = 4.530608e+01   |
| 10000   | DeltaV = 5.015437e-05    | uLB_maks = 1.539607e-01  | OutVMean = 3.309176e-04   | InVMean = 9.002494e-04   | DMF = 6.324157e+01   | DMF_LB = 6.324157e+01   | MLUPS = 4.530823e+01   |
| 11000   | DeltaV = 4.448292e-05    | uLB_maks = 1.541767e-01  | OutVMean = 3.690324e-04   | InVMean = 9.002494e-04   | DMF = 5.900776e+01   | DMF_LB = 5.900776e+01   | MLUPS = 4.531116e+01   |
| 12000   | DeltaV = 3.969818e-05    | uLB_maks = 1.543764e-01  | OutVMean = 4.046162e-04   | InVMean = 9.002494e-04   | DMF = 5.505510e+01   | DMF_LB = 5.505510e+01   | MLUPS = 4.530836e+01   |
| 13000   | DeltaV = 3.560738e-05    | uLB_maks = 1.545618e-01  | OutVMean = 4.378077e-04   | InVMean = 9.002494e-04   | DMF = 5.136817e+01   | DMF_LB = 5.136817e+01   | MLUPS = 4.530643e+01   |
| 14000   | DeltaV = 3.207311e-05    | uLB_maks = 1.547426e-01  | OutVMean = 4.687554e-04   | InVMean = 9.002494e-04   | DMF = 4.793049e+01   | DMF_LB = 4.793049e+01   | MLUPS = 4.527925e+01   |
| 15000   | DeltaV = 2.899368e-05    | uLB_maks = 1.549110e-01  | OutVMean = 4.976071e-04   | InVMean = 9.002494e-04   | DMF = 4.472565e+01   | DMF_LB = 4.472565e+01   | MLUPS = 4.531593e+01   |
| 16000   | DeltaV = 2.629156e-05    | uLB_maks = 1.550679e-01  | OutVMean = 5.245045e-04   | InVMean = 9.002494e-04   | DMF = 4.173787e+01   | DMF_LB = 4.173787e+01   | MLUPS = 4.530741e+01   |
| 17000   | DeltaV = 2.390621e-05    | uLB_maks = 1.552152e-01  | OutVMean = 5.495816e-04   | InVMean = 9.002494e-04   | DMF = 3.895230e+01   | DMF_LB = 3.895230e+01   | MLUPS = 4.530888e+01   |
| 18000   | DeltaV = 2.178953e-05    | uLB_maks = 1.553580e-01  | OutVMean = 5.729638e-04   | InVMean = 9.002494e-04   | DMF = 3.635499e+01   | DMF_LB = 3.635499e+01   | MLUPS = 4.530831e+01   |
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| Step    | Sc velocity residue, [-] | Maximal LB velocity, [-] | M outlet velocity, [m/s]  | M inlet velocity, [m/s]  | Error mass flow, [%] | Error mass flow LB, [%] | MLUPS, [-]            |
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| 19000   | DeltaV = 1.990267e-05    | uLB_maks = 1.554913e-01  | OutVMean = 5.947680e-04   | InVMean = 9.002494e-04   | DMF = 3.393297e+01   | DMF_LB = 3.393297e+01   | MLUPS = 4.531227e+01   |
| 20000   | DeltaV = 1.821388e-05    | uLB_maks = 1.556155e-01  | OutVMean = 6.151030e-04   | InVMean = 9.002494e-04   | DMF = 3.167416e+01   | DMF_LB = 3.167416e+01   | MLUPS = 4.530537e+01   |
| 21000   | DeltaV = 1.669692e-05    | uLB_maks = 1.557329e-01  | OutVMean = 6.340698e-04   | InVMean = 9.002494e-04   | DMF = 2.956732e+01   | DMF_LB = 2.956732e+01   | MLUPS = 4.531025e+01   |
| 22000   | DeltaV = 1.532989e-05    | uLB_maks = 1.558460e-01  | OutVMean = 6.517623e-04   | InVMean = 9.002494e-04   | DMF = 2.760203e+01   | DMF_LB = 2.760203e+01   | MLUPS = 4.531070e+01   |
| 23000   | DeltaV = 1.409440e-05    | uLB_maks = 1.559515e-01  | OutVMean = 6.682677e-04   | InVMean = 9.002494e-04   | DMF = 2.576861e+01   | DMF_LB = 2.576861e+01   | MLUPS = 4.531252e+01   |
| 24000   | DeltaV = 1.297485e-05    | uLB_maks = 1.560499e-01  | OutVMean = 6.836671e-04   | InVMean = 9.002494e-04   | DMF = 2.405803e+01   | DMF_LB = 2.405803e+01   | MLUPS = 4.531333e+01   |
| 25000   | DeltaV = 1.195792e-05    | uLB_maks = 1.561433e-01  | OutVMean = 6.980359e-04   | InVMean = 9.002494e-04   | DMF = 2.246194e+01   | DMF_LB = 2.246194e+01   | MLUPS = 4.531022e+01   |
| 26000   | DeltaV = 1.103221e-05    | uLB_maks = 1.562328e-01  | OutVMean = 7.114441e-04   | InVMean = 9.002494e-04   | DMF = 2.097256e+01   | DMF_LB = 2.097256e+01   | MLUPS = 4.530911e+01   |
| 27000   | DeltaV = 1.018787e-05    | uLB_maks = 1.563163e-01  | OutVMean = 7.239569e-04   | InVMean = 9.002494e-04   | DMF = 1.958263e+01   | DMF_LB = 1.958263e+01   | MLUPS = 4.531201e+01   |
| 28000   | DeltaV = 9.416349e-06    | uLB_maks = 1.563943e-01  | OutVMean = 7.356349e-04   | InVMean = 9.002494e-04   | DMF = 1.828544e+01   | DMF_LB = 1.828544e+01   | MLUPS = 4.531219e+01   |
| 29000   | DeltaV = 8.710199e-06    | uLB_maks = 1.564684e-01  | OutVMean = 7.465345e-04   | InVMean = 9.002494e-04   | DMF = 1.707471e+01   | DMF_LB = 1.707471e+01   | MLUPS = 4.530923e+01   |
| 30000   | DeltaV = 8.062902e-06    | uLB_maks = 1.565392e-01  | OutVMean = 7.567082e-04   | InVMean = 9.002494e-04   | DMF = 1.594461e+01   | DMF_LB = 1.594461e+01   | MLUPS = 4.531048e+01   |
| 31000   | DeltaV = 7.468724e-06    | uLB_maks = 1.566052e-01  | OutVMean = 7.662049e-04   | InVMean = 9.002494e-04   | DMF = 1.488971e+01   | DMF_LB = 1.488971e+01   | MLUPS = 4.530480e+01   |
| 32000   | DeltaV = 6.922608e-06    | uLB_maks = 1.566669e-01  | OutVMean = 7.750701e-04   | InVMean = 9.002494e-04   | DMF = 1.390496e+01   | DMF_LB = 1.390496e+01   | MLUPS = 4.529848e+01   |
| 33000   | DeltaV = 6.420076e-06    | uLB_maks = 1.567257e-01  | OutVMean = 7.833463e-04   | InVMean = 9.002494e-04   | DMF = 1.298564e+01   | DMF_LB = 1.298564e+01   | MLUPS = 4.530887e+01   |
| 34000   | DeltaV = 5.957145e-06    | uLB_maks = 1.567816e-01  | OutVMean = 7.910728e-04   | InVMean = 9.002494e-04   | DMF = 1.212737e+01   | DMF_LB = 1.212737e+01   | MLUPS = 4.530600e+01   |
| 35000   | DeltaV = 5.530268e-06    | uLB_maks = 1.568338e-01  | OutVMean = 7.982866e-04   | InVMean = 9.002494e-04   | DMF = 1.132606e+01   | DMF_LB = 1.132606e+01   | MLUPS = 4.530679e+01   |
| 36000   | DeltaV = 5.136273e-06    | uLB_maks = 1.568826e-01  | OutVMean = 8.050219e-04   | InVMean = 9.002494e-04   | DMF = 1.057790e+01   | DMF_LB = 1.057790e+01   | MLUPS = 4.530467e+01   |
| 37000   | DeltaV = 4.772317e-06    | uLB_maks = 1.569291e-01  | OutVMean = 8.113107e-04   | InVMean = 9.002494e-04   | DMF = 9.879342e+00   | DMF_LB = 9.879342e+00   | MLUPS = 4.530568e+01   |
| 38000   | DeltaV = 4.435843e-06    | uLB_maks = 1.569733e-01  | OutVMean = 8.171828e-04   | InVMean = 9.002494e-04   | DMF = 9.227067e+00   | DMF_LB = 9.227067e+00   | MLUPS = 4.530175e+01   |
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| Step    | Sc velocity residue, [-] | Maximal LB velocity, [-] | M outlet velocity, [m/s]  | M inlet velocity, [m/s]  | Error mass flow, [%] | Error mass flow LB, [%] | MLUPS, [-]            |
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| 39000   | DeltaV = 4.124548e-06    | uLB_maks = 1.570145e-01  | OutVMean = 8.226660e-04   | InVMean = 9.002494e-04   | DMF = 8.617993e+00   | DMF_LB = 8.617993e+00   | MLUPS = 4.530955e+01   |
| 40000   | DeltaV = 3.836354e-06    | uLB_maks = 1.570530e-01  | OutVMean = 8.277862e-04   | InVMean = 9.002494e-04   | DMF = 8.049241e+00   | DMF_LB = 8.049241e+00   | MLUPS = 4.530566e+01   |
| 41000   | DeltaV = 3.569378e-06    | uLB_maks = 1.570898e-01  | OutVMean = 8.325675e-04   | InVMean = 9.002494e-04   | DMF = 7.518127e+00   | DMF_LB = 7.518127e+00   | MLUPS = 4.530648e+01   |
| 42000   | DeltaV = 3.321915e-06    | uLB_maks = 1.571247e-01  | OutVMean = 8.370326e-04   | InVMean = 9.002494e-04   | DMF = 7.022146e+00   | DMF_LB = 7.022146e+00   | MLUPS = 4.530834e+01   |
| 43000   | DeltaV = 3.092415e-06    | uLB_maks = 1.571572e-01  | OutVMean = 8.412024e-04   | InVMean = 9.002494e-04   | DMF = 6.558964e+00   | DMF_LB = 6.558964e+00   | MLUPS = 4.530690e+01   |
| 44000   | DeltaV = 2.879466e-06    | uLB_maks = 1.571876e-01  | OutVMean = 8.450965e-04   | InVMean = 9.002494e-04   | DMF = 6.126401e+00   | DMF_LB = 6.126401e+00   | MLUPS = 4.530268e+01   |
| 45000   | DeltaV = 2.681784e-06    | uLB_maks = 1.572166e-01  | OutVMean = 8.487333e-04   | InVMean = 9.002494e-04   | DMF = 5.722424e+00   | DMF_LB = 5.722424e+00   | MLUPS = 4.530418e+01   |
| 46000   | DeltaV = 2.498194e-06    | uLB_maks = 1.572441e-01  | OutVMean = 8.521298e-04   | InVMean = 9.002494e-04   | DMF = 5.345138e+00   | DMF_LB = 5.345138e+00   | MLUPS = 4.530956e+01   |
| 47000   | DeltaV = 2.327625e-06    | uLB_maks = 1.572697e-01  | OutVMean = 8.553020e-04   | InVMean = 9.002494e-04   | DMF = 4.992771e+00   | DMF_LB = 4.992771e+00   | MLUPS = 4.530635e+01   |
| 48000   | DeltaV = 2.169092e-06    | uLB_maks = 1.572937e-01  | OutVMean = 8.582647e-04   | InVMean = 9.002494e-04   | DMF = 4.663673e+00   | DMF_LB = 4.663673e+00   | MLUPS = 4.531168e+01   |
| 49000   | DeltaV = 2.021695e-06    | uLB_maks = 1.573166e-01  | OutVMean = 8.610318e-04   | InVMean = 9.002494e-04   | DMF = 4.356301e+00   | DMF_LB = 4.356301e+00   | MLUPS = 4.531113e+01   |
| 50000   | DeltaV = 1.884609e-06    | uLB_maks = 1.573382e-01  | OutVMean = 8.636163e-04   | InVMean = 9.002494e-04   | DMF = 4.069217e+00   | DMF_LB = 4.069217e+00   | MLUPS = 4.530802e+01   |
| 51000   | DeltaV = 1.757073e-06    | uLB_maks = 1.573584e-01  | OutVMean = 8.660302e-04   | InVMean = 9.002494e-04   | DMF = 3.801078e+00   | DMF_LB = 3.801078e+00   | MLUPS = 4.531024e+01   |
| 52000   | DeltaV = 1.638389e-06    | uLB_maks = 1.573772e-01  | OutVMean = 8.682849e-04   | InVMean = 9.002494e-04   | DMF = 3.550631e+00   | DMF_LB = 3.550631e+00   | MLUPS = 4.530885e+01   |
| 53000   | DeltaV = 1.527914e-06    | uLB_maks = 1.573953e-01  | OutVMean = 8.703908e-04   | InVMean = 9.002494e-04   | DMF = 3.316705e+00   | DMF_LB = 3.316705e+00   | MLUPS = 4.531206e+01   |
| 54000   | DeltaV = 1.425055e-06    | uLB_maks = 1.574123e-01  | OutVMean = 8.723578e-04   | InVMean = 9.002494e-04   | DMF = 3.098208e+00   | DMF_LB = 3.098208e+00   | MLUPS = 4.530638e+01   |
| 55000   | DeltaV = 1.329265e-06    | uLB_maks = 1.574281e-01  | OutVMean = 8.741951e-04   | InVMean = 9.002494e-04   | DMF = 2.894120e+00   | DMF_LB = 2.894120e+00   | MLUPS = 4.530713e+01   |
| 56000   | DeltaV = 1.240040e-06    | uLB_maks = 1.574430e-01  | OutVMean = 8.759113e-04   | InVMean = 9.002494e-04   | DMF = 2.703490e+00   | DMF_LB = 2.703490e+00   | MLUPS = 4.530550e+01   |
| 57000   | DeltaV = 1.156914e-06    | uLB_maks = 1.574572e-01  | OutVMean = 8.775143e-04   | InVMean = 9.002494e-04   | DMF = 2.525427e+00   | DMF_LB = 2.525427e+00   | MLUPS = 4.530497e+01   |
| 58000   | DeltaV = 1.079455e-06    | uLB_maks = 1.574705e-01  | OutVMean = 8.790116e-04   | InVMean = 9.002494e-04   | DMF = 2.359102e+00   | DMF_LB = 2.359102e+00   | MLUPS = 4.530709e+01   |
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| Step    | Sc velocity residue, [-] | Maximal LB velocity, [-] | M outlet velocity, [m/s]  | M inlet velocity, [m/s]  | Error mass flow, [%] | Error mass flow LB, [%] | MLUPS, [-]            |
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| 59000   | DeltaV = 1.007265e-06    | uLB_maks = 1.574830e-01  | OutVMean = 8.804103e-04   | InVMean = 9.002494e-04   | DMF = 2.203740e+00   | DMF_LB = 2.203740e+00   | MLUPS = 4.530625e+01   |
| 60000   | DeltaV = 9.399754e-07    | uLB_maks = 1.574947e-01  | OutVMean = 8.817167e-04   | InVMean = 9.002494e-04   | DMF = 2.058617e+00   | DMF_LB = 2.058617e+00   | MLUPS = 4.530777e+01   |
| 61000   | DeltaV = 8.772434e-07    | uLB_maks = 1.575058e-01  | OutVMean = 8.829371e-04   | InVMean = 9.002494e-04   | DMF = 1.923058e+00   | DMF_LB = 1.923058e+00   | MLUPS = 4.531028e+01   |
| 62000   | DeltaV = 8.187526e-07    | uLB_maks = 1.575163e-01  | OutVMean = 8.840771e-04   | InVMean = 9.002494e-04   | DMF = 1.796431e+00   | DMF_LB = 1.796431e+00   | MLUPS = 4.530407e+01   |
| 63000   | DeltaV = 7.642092e-07    | uLB_maks = 1.575261e-01  | OutVMean = 8.851419e-04   | InVMean = 9.002494e-04   | DMF = 1.678147e+00   | DMF_LB = 1.678147e+00   | MLUPS = 4.529702e+01   |
| 64000   | DeltaV = 7.133407e-07    | uLB_maks = 1.575353e-01  | OutVMean = 8.861366e-04   | InVMean = 9.002494e-04   | DMF = 1.567655e+00   | DMF_LB = 1.567655e+00   | MLUPS = 4.530500e+01   |
| 65000   | DeltaV = 6.658942e-07    | uLB_maks = 1.575440e-01  | OutVMean = 8.870658e-04   | InVMean = 9.002494e-04   | DMF = 1.464443e+00   | DMF_LB = 1.464443e+00   | MLUPS = 4.530385e+01   |
| 66000   | DeltaV = 6.216349e-07    | uLB_maks = 1.575523e-01  | OutVMean = 8.879337e-04   | InVMean = 9.002494e-04   | DMF = 1.368029e+00   | DMF_LB = 1.368029e+00   | MLUPS = 4.530410e+01   |
| 67000   | DeltaV = 5.803446e-07    | uLB_maks = 1.575600e-01  | OutVMean = 8.887445e-04   | InVMean = 9.002494e-04   | DMF = 1.277966e+00   | DMF_LB = 1.277966e+00   | MLUPS = 4.530287e+01   |
| 68000   | DeltaV = 5.418207e-07    | uLB_maks = 1.575671e-01  | OutVMean = 8.895019e-04   | InVMean = 9.002494e-04   | DMF = 1.193834e+00   | DMF_LB = 1.193834e+00   | MLUPS = 4.530472e+01   |
| 69000   | DeltaV = 5.058748e-07    | uLB_maks = 1.575740e-01  | OutVMean = 8.902094e-04   | InVMean = 9.002494e-04   | DMF = 1.115243e+00   | DMF_LB = 1.115243e+00   | MLUPS = 4.530074e+01   |
| 70000   | DeltaV = 4.723317e-07    | uLB_maks = 1.575805e-01  | OutVMean = 8.908704e-04   | InVMean = 9.002494e-04   | DMF = 1.041828e+00   | DMF_LB = 1.041828e+00   | MLUPS = 4.530652e+01   |
| 71000   | DeltaV = 4.410285e-07    | uLB_maks = 1.576246e-01  | OutVMean = 8.914878e-04   | InVMean = 9.002494e-04   | DMF = 9.732476e-01   | DMF_LB = 9.732476e-01   | MLUPS = 4.530361e+01   |
| 72000   | DeltaV = 4.118135e-07    | uLB_maks = 1.577264e-01  | OutVMean = 8.920645e-04   | InVMean = 9.002494e-04   | DMF = 9.091829e-01   | DMF_LB = 9.091829e-01   | MLUPS = 4.530664e+01   |
| 73000   | DeltaV = 3.845459e-07    | uLB_maks = 1.578215e-01  | OutVMean = 8.926033e-04   | InVMean = 9.002494e-04   | DMF = 8.493367e-01   | DMF_LB = 8.493367e-01   | MLUPS = 4.530791e+01   |
| 74000   | DeltaV = 3.590941e-07    | uLB_maks = 1.579104e-01  | OutVMean = 8.931066e-04   | InVMean = 9.002494e-04   | DMF = 7.934309e-01   | DMF_LB = 7.934309e-01   | MLUPS = 4.530184e+01   |
| 75000   | DeltaV = 3.353360e-07    | uLB_maks = 1.579934e-01  | OutVMean = 8.935767e-04   | InVMean = 9.002494e-04   | DMF = 7.412059e-01   | DMF_LB = 7.412059e-01   | MLUPS = 4.530386e+01   |
| 76000   | DeltaV = 3.131576e-07    | uLB_maks = 1.580709e-01  | OutVMean = 8.940159e-04   | InVMean = 9.002494e-04   | DMF = 6.924194e-01   | DMF_LB = 6.924194e-01   | MLUPS = 4.530911e+01   |
| 77000   | DeltaV = 2.924530e-07    | uLB_maks = 1.581433e-01  | OutVMean = 8.944262e-04   | InVMean = 9.002494e-04   | DMF = 6.468448e-01   | DMF_LB = 6.468448e-01   | MLUPS = 4.531296e+01   |
| 78000   | DeltaV = 2.731233e-07    | uLB_maks = 1.582110e-01  | OutVMean = 8.948095e-04   | InVMean = 9.002494e-04   | DMF = 6.042705e-01   | DMF_LB = 6.042705e-01   | MLUPS = 4.531051e+01   |
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| Step    | Sc velocity residue, [-] | Maximal LB velocity, [-] | M outlet velocity, [m/s]  | M inlet velocity, [m/s]  | Error mass flow, [%] | Error mass flow LB, [%] | MLUPS, [-]            |
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| 79000   | DeltaV = 2.550765e-07    | uLB_maks = 1.582742e-01  | OutVMean = 8.951675e-04   | InVMean = 9.002494e-04   | DMF = 5.644989e-01   | DMF_LB = 5.644989e-01   | MLUPS = 4.530872e+01   |
| 80000   | DeltaV = 2.382267e-07    | uLB_maks = 1.583332e-01  | OutVMean = 8.955020e-04   | InVMean = 9.002494e-04   | DMF = 5.273455e-01   | DMF_LB = 5.273455e-01   | MLUPS = 4.530381e+01   |
| 81000   | DeltaV = 2.224939e-07    | uLB_maks = 1.583883e-01  | OutVMean = 8.958144e-04   | InVMean = 9.002494e-04   | DMF = 4.926379e-01   | DMF_LB = 4.926379e-01   | MLUPS = 4.530011e+01   |
| 82000   | DeltaV = 2.078037e-07    | uLB_maks = 1.584399e-01  | OutVMean = 8.961063e-04   | InVMean = 9.002494e-04   | DMF = 4.602150e-01   | DMF_LB = 4.602150e-01   | MLUPS = 4.530278e+01   |
| 83000   | DeltaV = 1.940864e-07    | uLB_maks = 1.584880e-01  | OutVMean = 8.963790e-04   | InVMean = 9.002494e-04   | DMF = 4.299262e-01   | DMF_LB = 4.299262e-01   | MLUPS = 4.530365e+01   |
| 84000   | DeltaV = 1.812772e-07    | uLB_maks = 1.585330e-01  | OutVMean = 8.966337e-04   | InVMean = 9.002494e-04   | DMF = 4.016313e-01   | DMF_LB = 4.016313e-01   | MLUPS = 4.530447e+01   |
```

```
| 85000   | DeltaV = 1.693157e-07    | uLB_maks = 1.585750e-01  | OutVMean = 8.968717e-04   | InVMean = 9.002494e-04   | DMF = 3.751987e-01    | DMF_LB = 3.751987e-01    | MLUPS = 4.530410e+01    |
| 86000   | DeltaV = 1.581455e-07    | uLB_maks = 1.586142e-01  | OutVMean = 8.970940e-04   | InVMean = 9.002494e-04   | DMF = 3.505060e-01    | DMF_LB = 3.505060e-01    | MLUPS = 4.530196e+01    |
| 87000   | DeltaV = 1.477140e-07    | uLB_maks = 1.586509e-01  | OutVMean = 8.973017e-04   | InVMean = 9.002494e-04   | DMF = 3.274386e-01    | DMF_LB = 3.274386e-01    | MLUPS = 4.529859e+01    |
| 88000   | DeltaV = 1.379721e-07    | uLB_maks = 1.586851e-01  | OutVMean = 8.974957e-04   | InVMean = 9.002494e-04   | DMF = 3.058894e-01    | DMF_LB = 3.058894e-01    | MLUPS = 4.530459e+01    |
| 89000   | DeltaV = 1.288741e-07    | uLB_maks = 1.587171e-01  | OutVMean = 8.976769e-04   | InVMean = 9.002494e-04   | DMF = 2.857586e-01    | DMF_LB = 2.857586e-01    | MLUPS = 4.530716e+01    |
| 90000   | DeltaV = 1.203771e-07    | uLB_maks = 1.587470e-01  | OutVMean = 8.978462e-04   | InVMean = 9.002494e-04   | DMF = 2.669527e-01    | DMF_LB = 2.669527e-01    | MLUPS = 4.530232e+01    |
| 91000   | DeltaV = 1.124414e-07    | uLB_maks = 1.587749e-01  | OutVMean = 8.980043e-04   | InVMean = 9.002494e-04   | DMF = 2.493845e-01    | DMF_LB = 2.493845e-01    | MLUPS = 4.530385e+01    |
| 92000   | DeltaV = 1.050297e-07    | uLB_maks = 1.588010e-01  | OutVMean = 8.981521e-04   | InVMean = 9.002494e-04   | DMF = 2.329726e-01    | DMF_LB = 2.329726e-01    | MLUPS = 4.530253e+01    |
| 93000   | DeltaV = 9.810733e-08    | uLB_maks = 1.588253e-01  | OutVMean = 8.982901e-04   | InVMean = 9.002494e-04   | DMF = 2.176409e-01    | DMF_LB = 2.176409e-01    | MLUPS = 4.515705e+01    |
| 94000   | DeltaV = 9.164189e-08    | uLB_maks = 1.588481e-01  | OutVMean = 8.984190e-04   | InVMean = 9.002494e-04   | DMF = 2.033182e-01    | DMF_LB = 2.033182e-01    | MLUPS = 4.528851e+01    |
| 95000   | DeltaV = 8.560312e-08    | uLB_maks = 1.588694e-01  | OutVMean = 8.985395e-04   | InVMean = 9.002494e-04   | DMF = 1.899381e-01    | DMF_LB = 1.899381e-01    | MLUPS = 4.530523e+01    |
| 96000   | DeltaV = 7.996279e-08    | uLB_maks = 1.588892e-01  | OutVMean = 8.986520e-04   | InVMean = 9.002494e-04   | DMF = 1.774386e-01    | DMF_LB = 1.774386e-01    | MLUPS = 4.523282e+01    |
| 97000   | DeltaV = 7.469455e-08    | uLB_maks = 1.589078e-01  | OutVMean = 8.987572e-04   | InVMean = 9.002494e-04   | DMF = 1.657617e-01    | DMF_LB = 1.657617e-01    | MLUPS = 4.531118e+01    |
| 98000   | DeltaV = 6.977379e-08    | uLB_maks = 1.589251e-01  | OutVMean = 8.988554e-04   | InVMean = 9.002494e-04   | DMF = 1.548533e-01    | DMF_LB = 1.548533e-01    | MLUPS = 4.530741e+01    |
================================================================================================================================================================================================
| Step    | Sc velocity residue, [-] | Maximal LB velocity, [-] | M outlet velocity, [m/s]  | M inlet velocity, [m/s]  | Error mass flow, [%] | Error mass flow LB, [%] | MLUPS, [-]              |
================================================================================================================================================================================================
| 99000   | DeltaV = 6.517755e-08    | uLB_maks = 1.589413e-01  | OutVMean = 8.989471e-04   | InVMean = 9.002494e-04   | DMF = 1.446628e-01    | DMF_LB = 1.446628e-01    | MLUPS = 4.523032e+01    |
| 100000  | DeltaV = 6.088437e-08    | uLB_maks = 1.589564e-01  | OutVMean = 8.990328e-04   | InVMean = 9.002494e-04   | DMF = 1.351430e-01    | DMF_LB = 1.351430e-01    | MLUPS = 4.530039e+01    |
| 101000  | DeltaV = 5.687424e-08    | uLB_maks = 1.589706e-01  | OutVMean = 8.991129e-04   | InVMean = 9.002494e-04   | DMF = 1.262496e-01    | DMF_LB = 1.262496e-01    | MLUPS = 4.531027e+01    |
| 102000  | DeltaV = 5.312847e-08    | uLB_maks = 1.589838e-01  | OutVMean = 8.991877e-04   | InVMean = 9.002494e-04   | DMF = 1.179415e-01    | DMF_LB = 1.179415e-01    | MLUPS = 4.523351e+01    |
| 103000  | DeltaV = 4.962959e-08    | uLB_maks = 1.589961e-01  | OutVMean = 8.992575e-04   | InVMean = 9.002494e-04   | DMF = 1.101802e-01    | DMF_LB = 1.101802e-01    | MLUPS = 4.530725e+01    |
| 104000  | DeltaV = 4.636131e-08    | uLB_maks = 1.590076e-01  | OutVMean = 8.993228e-04   | InVMean = 9.002494e-04   | DMF = 1.029296e-01    | DMF_LB = 1.029296e-01    | MLUPS = 4.530894e+01    |
| 105000  | DeltaV = 4.330841e-08    | uLB_maks = 1.590184e-01  | OutVMean = 8.993838e-04   | InVMean = 9.002494e-04   | DMF = 9.615618e-02    | DMF_LB = 9.615618e-02    | MLUPS = 4.523414e+01    |
================================================================================================================================================================================================
VTI file was saved successfully: ----> Data/Validation_tests/Case_MEDIUM_straight_square_channel/output/microflow_output105000.vti
================================================================================================================================================================================================

Solver statistics :
Computation time [s] = 1023.19      mean MLUPS = 45.301      min MLUPS = 45.1571      max MLUPS = 45.3159      Average memory usage [kB] = 223308


--------------------------------------------------------------------------------
Microflow 3D ***** Goodbye *****
--------------------------------------------------------------------------------
```

6. Computer configuration and operating system

```
System:    Kernel: 4.15.0-115-generic x86_64 bits: 64 gcc: 7.5.0
           Desktop: Cinnamon 3.8.9 (Gtk 3.22.30-1ubuntu4) dm: lightdm Distro:
Linux Mint 19 Tara
Machine:   Device: desktop Mobo: Micro-Star model: MAG Z390 TOMAHAWK (MS-7B18)
         v: 1.0 serial: N/A UEFI: American Megatrends v: 1.70 date: 12/27/2019
CPU:       8 core Intel Core i9-9900K (-MCP-) arch: Skylake rev.12 cache: 16384
         KB flags: (lm nx sse sse2 sse3 sse4_1 sse4_2 ssse3 vmx) bmips: 57600
           clock speeds: min/max: 800/5000 MHz
Graphics:  Card: NVIDIA GK110 [GeForce GTX TITAN]
Memory:    Size: 2 x 16384 MB, Type: DDR4, Type Detail: Synchronous, Speed: 3000
         MT/s
```

7. Performance

CPU info:
Maximum number of available CPU threads = 8
Number of CPU threads allocated for solver = 6

Computation time [s] = 1023.19 mean MLUPS = 45.301 min MLUPS = 45.1571 max MLUPS = 45.3159 Average memory usage [kB] = 223308

MLUPS = Million of Lattice nodes Updated Per Second

8. Termination

The computations were terminated when the mass flow error (the relative difference between inlet and outlet mass flow) for subsequent steps calculated reached the value lower than 0.1% (DMF and DMF_LB = 9.615618e-02). At the same time, the mean scaled residue of average velocity of fluid for subsequent steps were as low as DeltaV = 4.330841e-08.

9. Results of simulation

9.1 Pressure drop analysis

All calculations of pressure drop are collected in a spreadsheet Pressure_drop_calculations.ods located in a folder /Data/Validation_tests/Case_MEDIUM_straight_square_channel/description/
Results of pressure drop simulation were compared with theoretical values calculated from equation (1) for fully developed laminar flow and with values calculated from eq. (9) that also takes into account hydrodynamic developing region at the beginning of channel. The Poiseuille (Po) number for square duct was calculated according to formula proposed by Serwinski and Delplace. The *Po* number for developing zone was calculated from eq. (15) with parameters proposed by Kandlikar and self-determined values of parameters of approximating equation (15), based on the values presented in the table 3.

In Table 5, there are compared minimal, maximal and average simulation errors. The relative errors were calculated as a difference between predicted in simulation values of gauge pressure P_{sim} and values calculated from analytical solution of eq. (1) P_{an} for subsequent points along a channel.

$$err[\%]=\left|\frac{P_{an}-P_{\mathrm{sim}}}{P_{an}}\right|*100 \quad (19)$$

Table 5. Simulation errors

	Po from Serwinski	*Po* from Delplace	Po_{app} from eq. 15 (original values)	Po_{app} from eq. 15 (self-determined values)
Average error [%]:	0.180637	0.341660	0.376121	0.331497
Min. error [%]:	0.027420	0.032209	0.032209	0.073712
Max error [%]:	5.825271	5.995823	5.995823	5.951819

The pressure profile along a channel and relative errors are also compared on following figures.

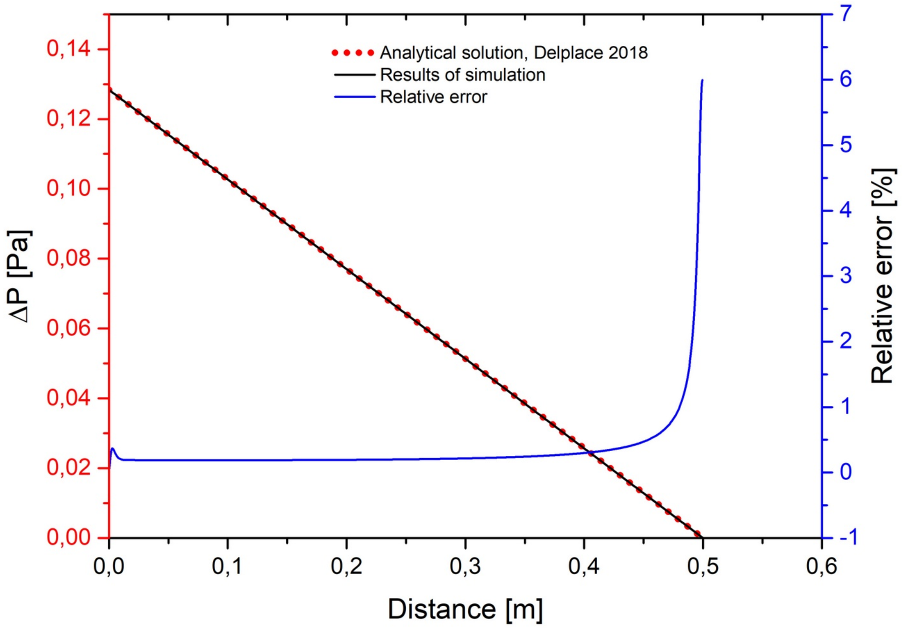

Figure 6. Pressure drop along the channel and relative error of prediction. Po value from Delplace 2018

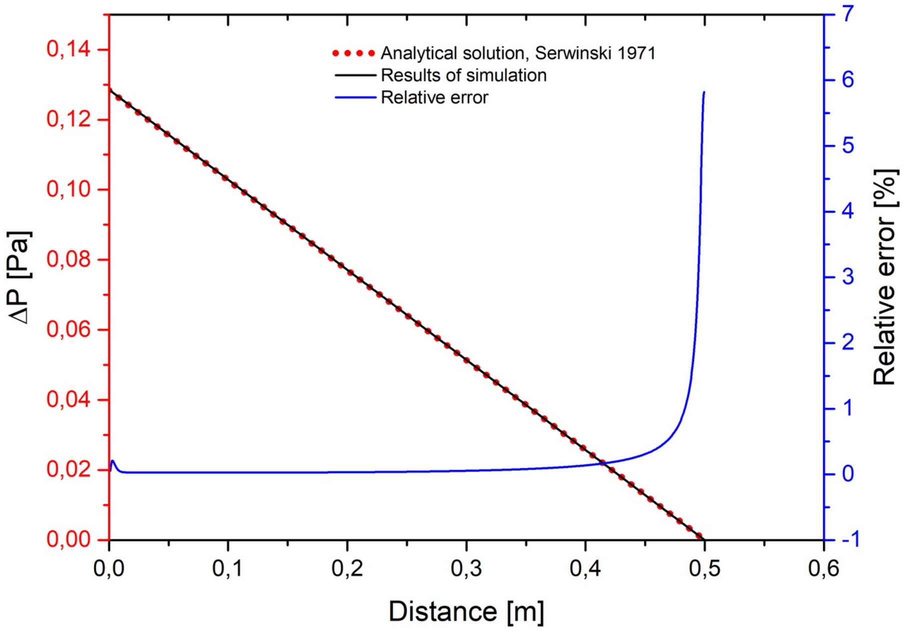

Figure 7. Pressure drop along the channel and relative error of prediction. Po value from Serwinski 1971

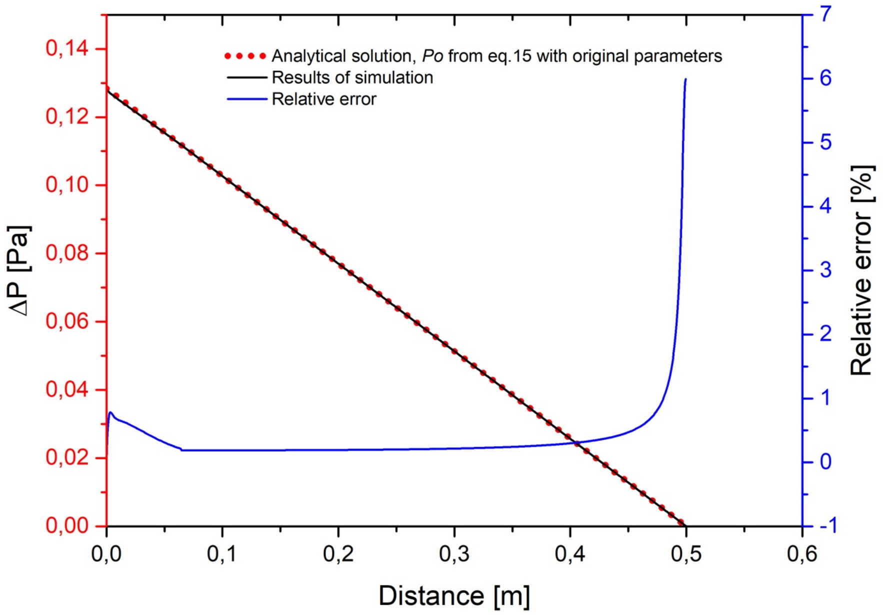

Figure 8. Pressure drop along the channel and relative error of prediction. Po value from eq. 15 with original parameters

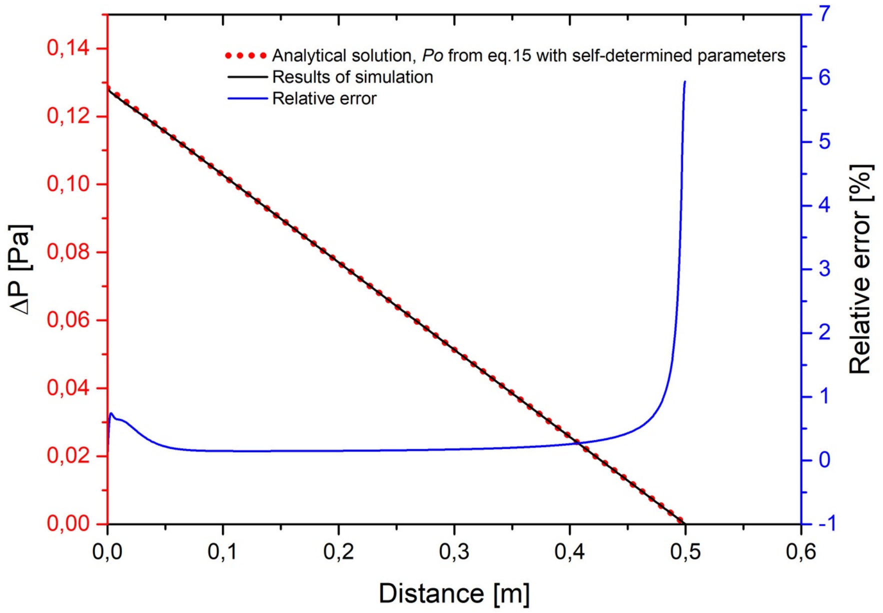

Figure 9. Pressure drop along the channel and relative error of prediction. Po value from eq. 15 with self-determined parameters

8.2 Velocity profile analysis

The dimensionless velocity profile in a square duct was calculated from eq. (17) and transferred to physical velocity according to eq. (18). All calculations were performed in Matlab. The source code square_profile_v2.m is attached in the folder:
/Data/Validation_tests/Case_MEDIUM_straight_square_channel/description/matlab/
Below, there are presented results for cross-section of channel at a half length of channel. Results of simulation are compared with algebraic solution of model and relative error was calculated for each node.

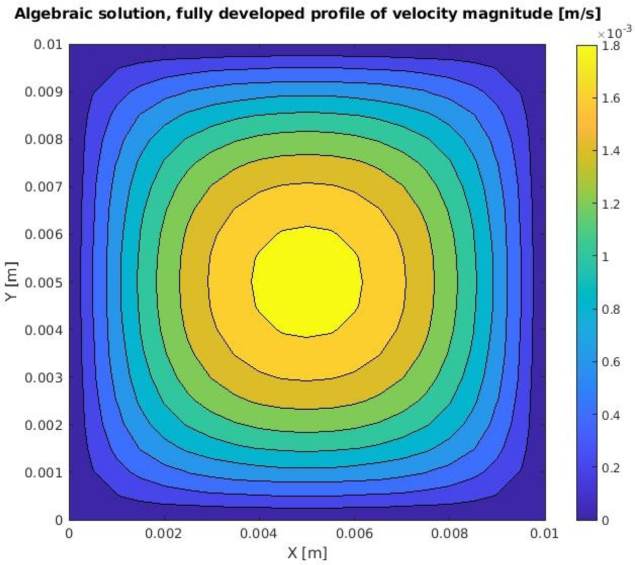

Figure 10. Analytical solution of model, fully developed velocity profile from eq. 17

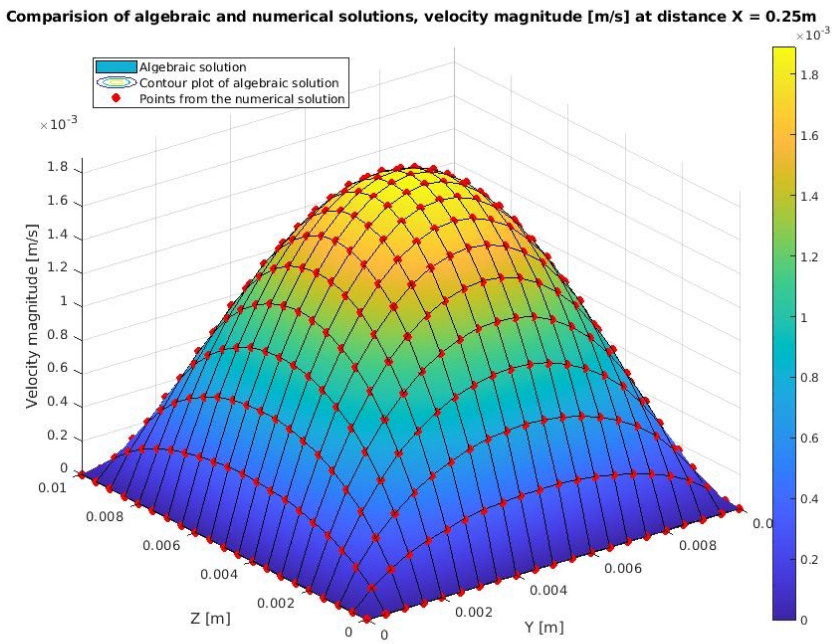

Figure 11. Comparison of algebraic and numeric solution of model. Red dots – results of simulation, surface – results of analytical solution.

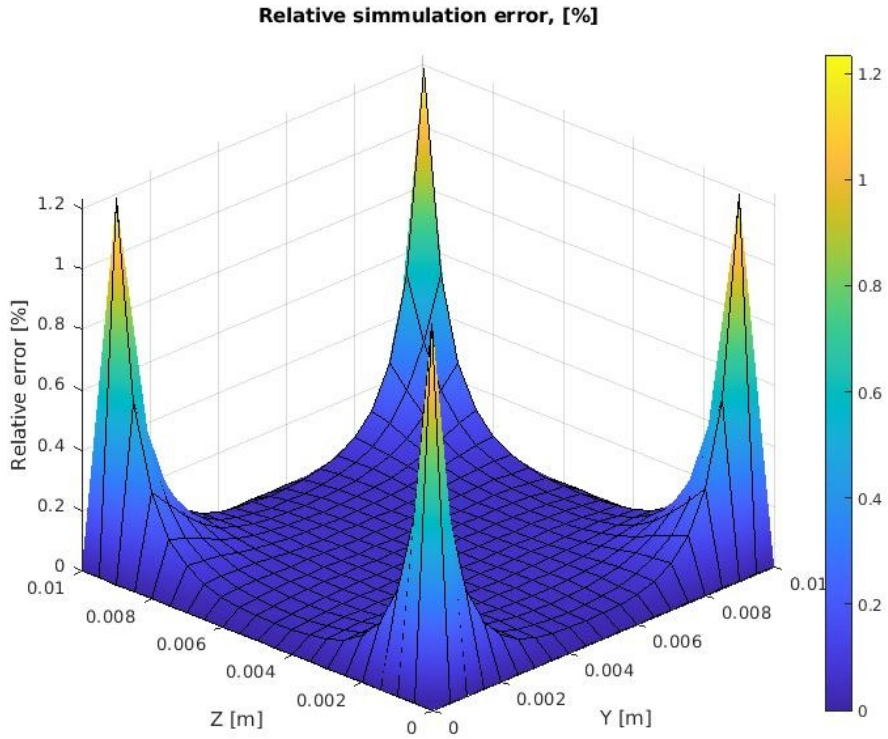

Figure 12. Relative simulation error distribution

Table 6. Simulation errors of velocity for calculations terminated at DMF = 9.615618e-02 %

Distance from the inlet, [m]	Mean error [%]	Median value of error [%]	Maximal error [%]	Minimal error [%]
0.01	0.46435	0.49089	0.99304	0.0048893
0.05	0.054942	0.0068117	1.1829	0.0001191
0.1	0.066323	0.017117	1.1971	0.0015823
0.2	0.092946	0.043677	1.2239	0.022795
0.25	0.10431	0.055629	1.2346	0.033979
0.3	0.11397	0.064924	1.2444	0.043672
0.4	0.12737	0.078868	1.2568	0.057093
0.45	0.13077	0.082169	1.2595	0.060324
0.5	2.0809	1.0883	9.2075	0.066907

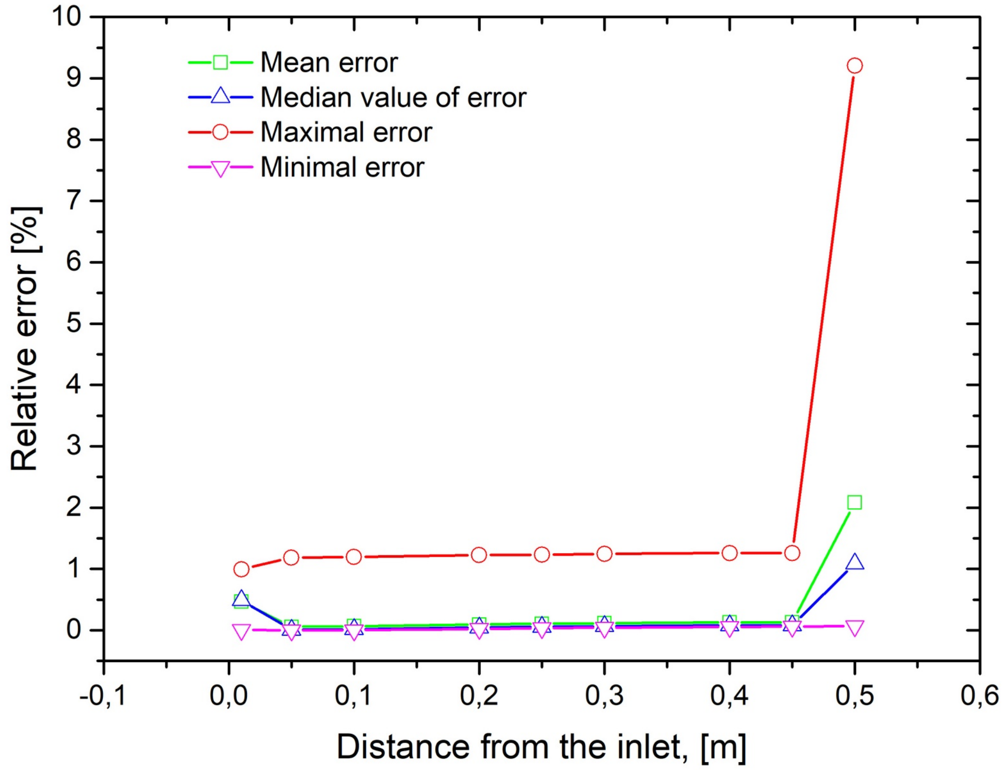

Figure 13. Relative simulation error of velocity along a channel length

Below, there is presented relative error distribution at outlet surface. There is observed strong impact of bounce-back nodes placed on edges.

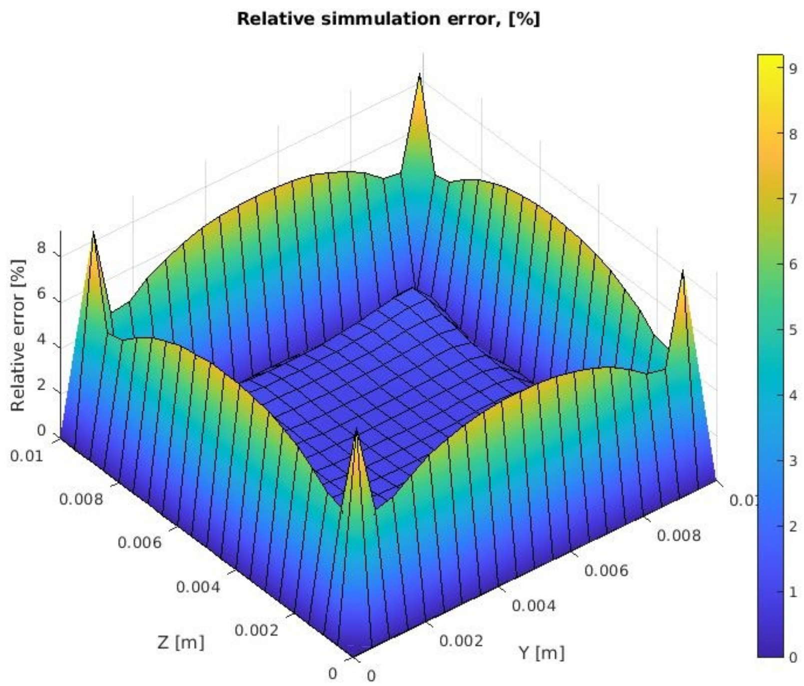

Figure 14. Relative simulation error of velocity at outlet surface

When termination condition MassFlowError_ErrM was lowered to 0.01, calculations were terminated after 139000 time steps. DMF/DMF_LB = 9.502872e-03 % and DeltaV = 4.277490e-09.

Solver statistics:
Computation time [s] = 1347.01 mean MLUPS = 45.5529 min MLUPS = 45.3258 max MLUPS = 45.5889 Average memory usage [kB] = 218232

Table 7. Simulation errors of velocity for calculations terminated at DMF = 9.502872e-03 %

Distance from the inlet, [m]	Mean error [%]	Median value of error [%]	Maximal error [%]	Minimal error [%]
0.01	0.46595	0.49421	0.99092	0.0080235
0.05	0.056802	0.014599	1.1678	8.2775e-05
0.1	0.056385	0.013268	1.1695	0.0018797
0.2	0.055686	0.010594	1.1722	0.00027605
0.25	0.055436	0.00975	1.1731	0.0012884
0.3	0.055219	0.0094624	1.174	0.00095962
0.4	0.054915	0.0081691	1.1749	0.00028379
0.45	0.054834	0.0081691	1.1758	0.00055417
0.5	2.1214	1.1759	9.1282	0.0015905

Literature:

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